Chapter 6: Work, Energy and Power
Thursday February 12th

- Discuss Mini Exam II
- Review: Work and Kinetic Energy
- Conservative and non-conservative forces
- Work and Potential Energy
- Conservation of Energy
- Calculus method for determining work
- Power
- As usual – *i*clicker, examples and demonstrations

Mini Exam III next Thursday

- Will cover LONCAPA #7-10 (Newton's laws and energy cons.)

Reading: up to page 97 in the text book (Ch. 6)

Work - Definition

Work W is the energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

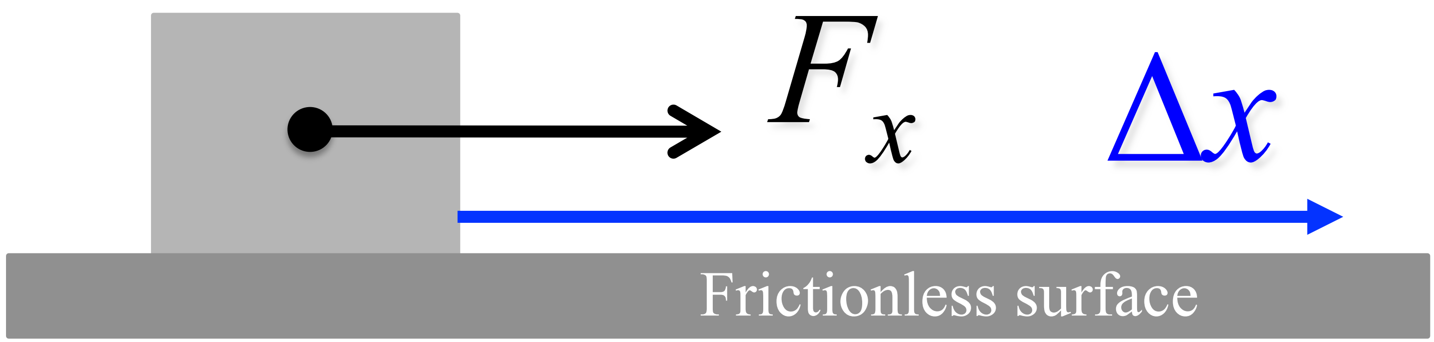
- There are only two relevant variables in one dimension: the force, F_x, and the displacement, Δx.

Definition: $W = F_x \Delta x$ [Units: N.m or Joule (J)]

F_x is the component of the force in the direction of the object's motion, and Δx is its displacement.

Kinetic Energy - Definition

$$K = \tfrac{1}{2} m v^2$$



$$v_f^2 = v_i^2 + 2 a_x \Delta x$$

$$\tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 = \tfrac{1}{2} m \times 2 a_x \Delta x$$

$$\Delta K = K_f - K_i = m a_x \Delta x = F_x \Delta x = W$$

Work-Kinetic Energy Theorem

$$\Delta K = K_f - K_i = W_{\text{net}}$$

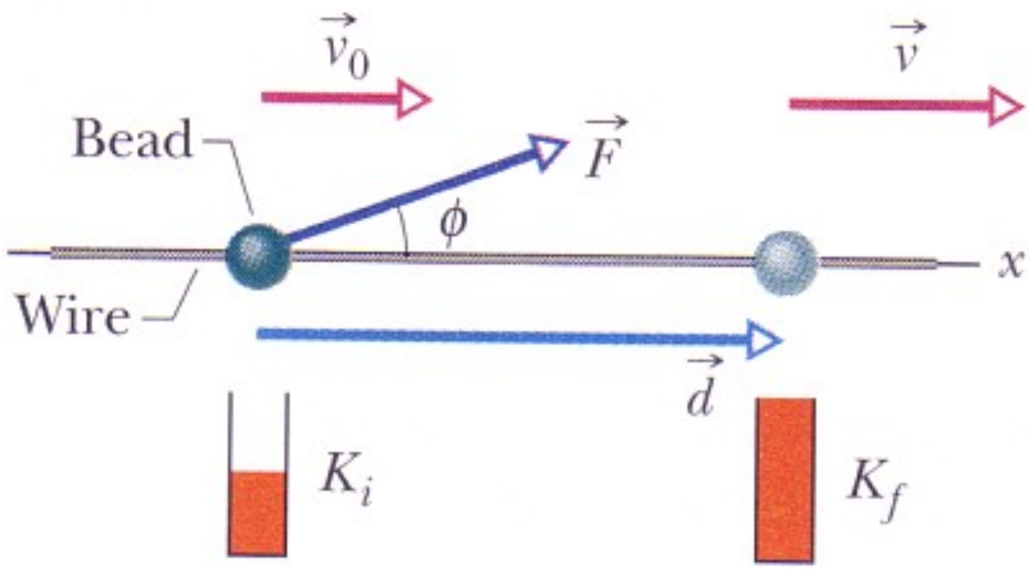
$$\begin{pmatrix} \text{change in the kinetic} \\ \text{energy of a particle} \end{pmatrix} = \begin{pmatrix} \text{net work done on} \\ \text{the particle} \end{pmatrix}$$

$$K_f = K_i + W_{\text{net}}$$

$$\begin{pmatrix} \text{kinetic energy after} \\ \text{the net work is done} \end{pmatrix} = \begin{pmatrix} \text{kinetic energy} \\ \text{before the net work} \end{pmatrix} + \begin{pmatrix} \text{the net} \\ \text{work done} \end{pmatrix}$$

More on Work

To calculate the **work** done on an object by a force during a displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work



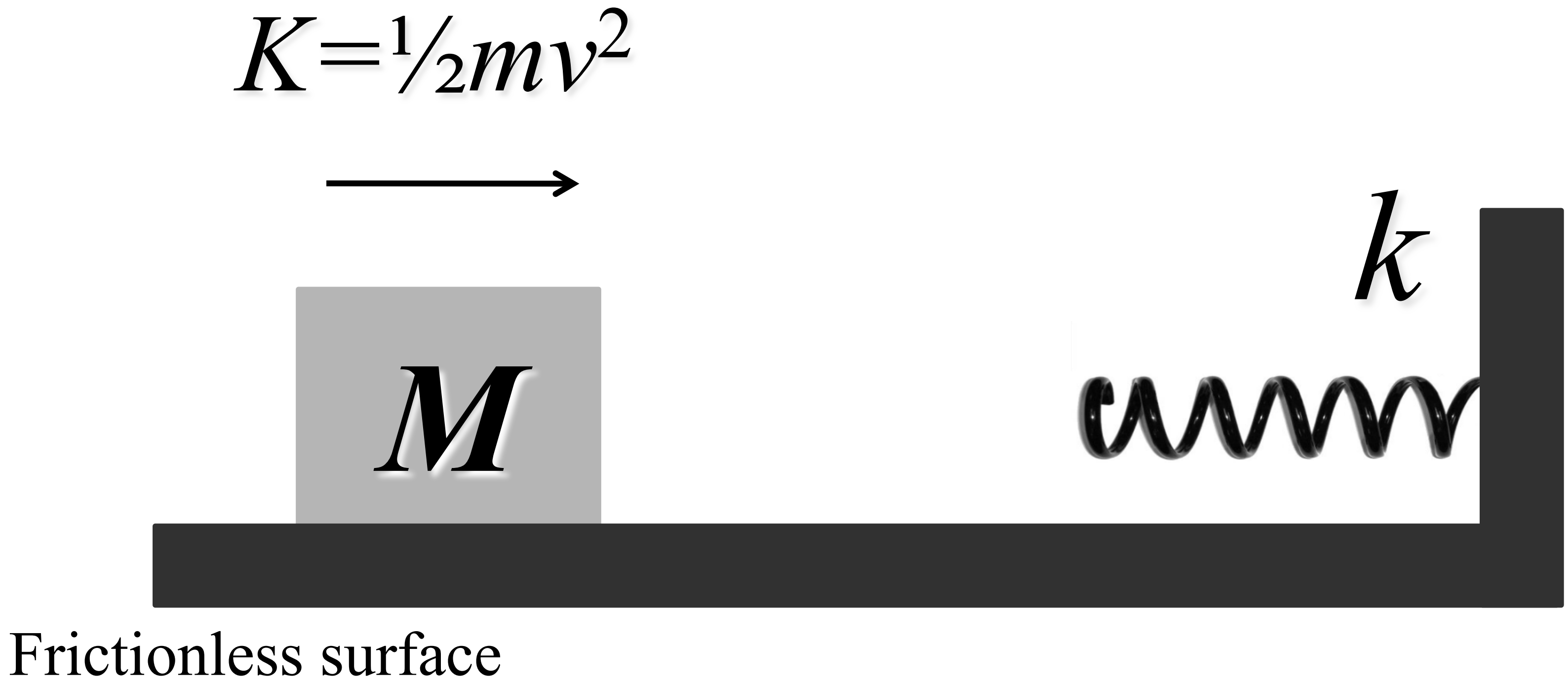
$$F_x = F\cos\phi$$

$$W = Fd\cos\phi$$

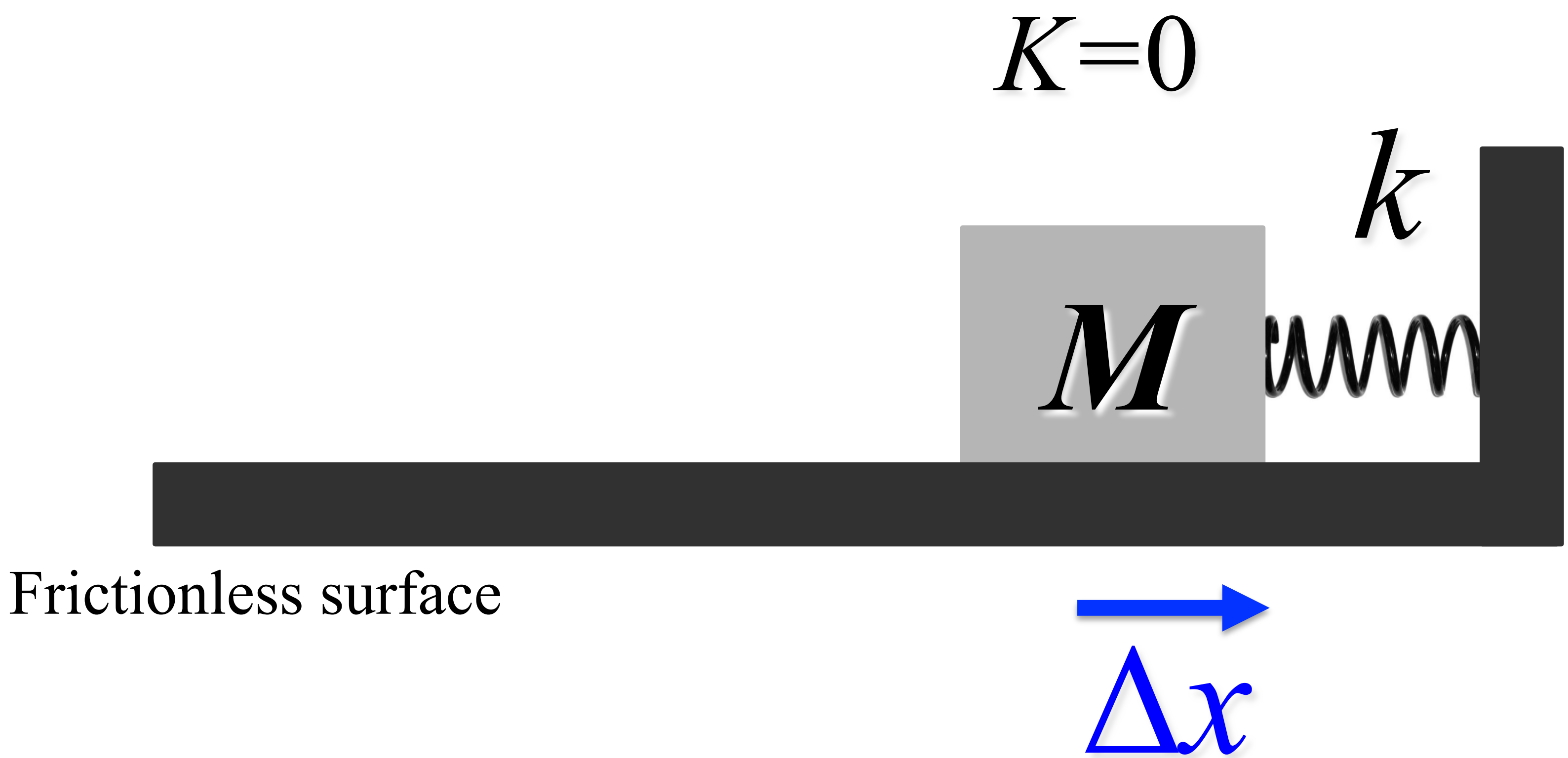
$$W = \vec{F}\cdot\vec{d}$$

- Caution: for all the equations we have derived so far, the force must be constant, and the object must be rigid.
- I will discuss variable forces later.

Work – More Examples



Work – More Examples

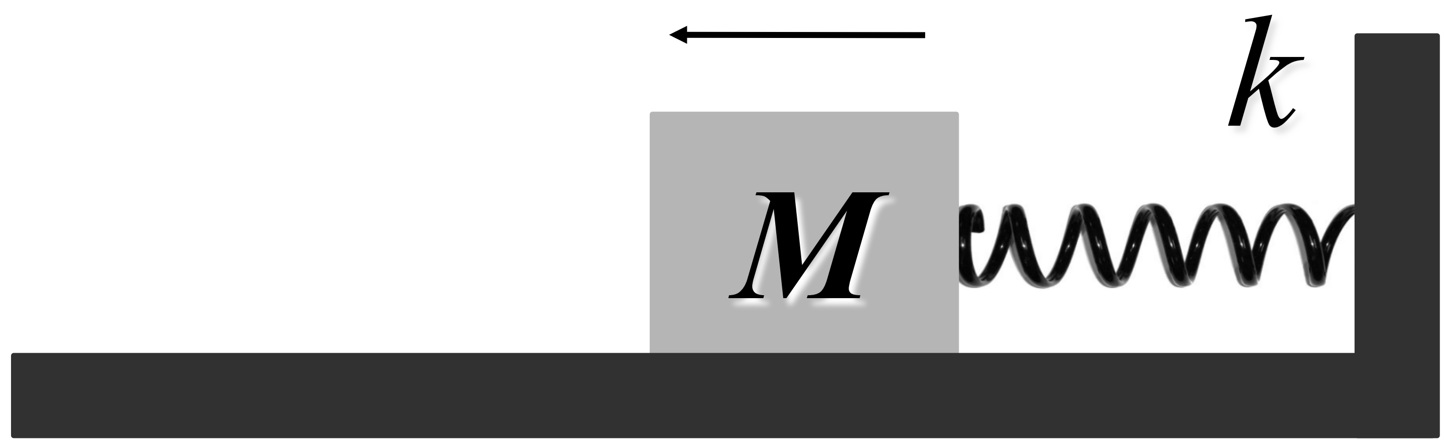


$$W_{\text{spr.}} = -\bar{F}\Delta x$$
$$= \Delta K$$

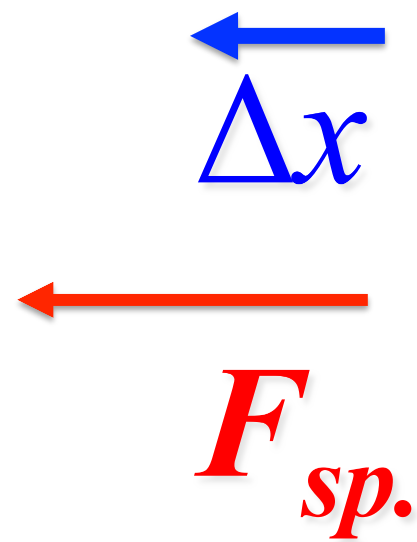
Work – More Examples

Kinetic energy is completely recovered: a ‘conservative’ force

$$K = \tfrac{1}{2} m v^2$$



Frictionless surface

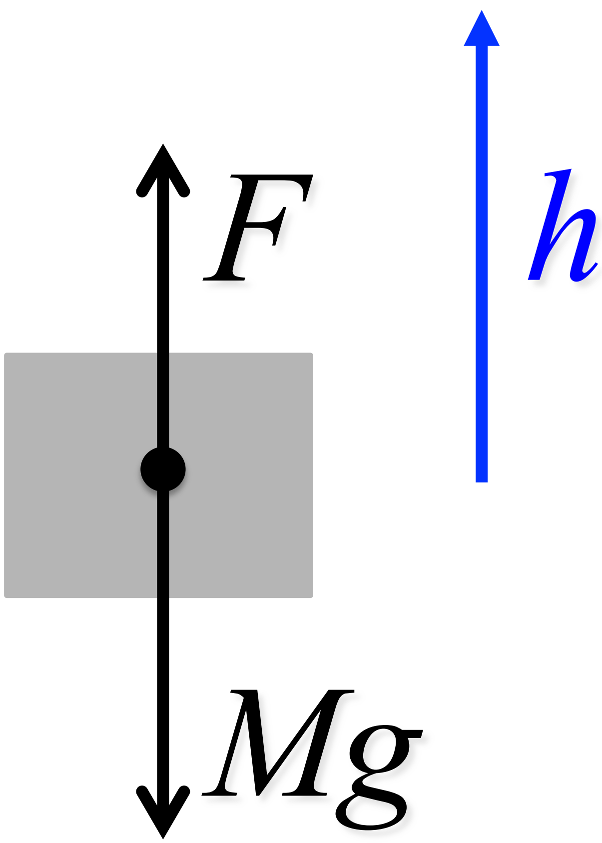


$$W_{spr.} = +\bar{F}\Delta x$$

$$= \Delta K$$

Work – More Examples

These two examples are similar – they both involve conservative forces
Examples: electrostatic (spring/elastic) forces, gravitational forces.



$$W_{\text{n.c.}} = Fh = +Mgh$$

$$W_{\text{cons.}} = W_g = -Mgh$$

$$W_{\text{net}} = W_{\text{n.c.}} + W_{\text{cons.}} = 0$$

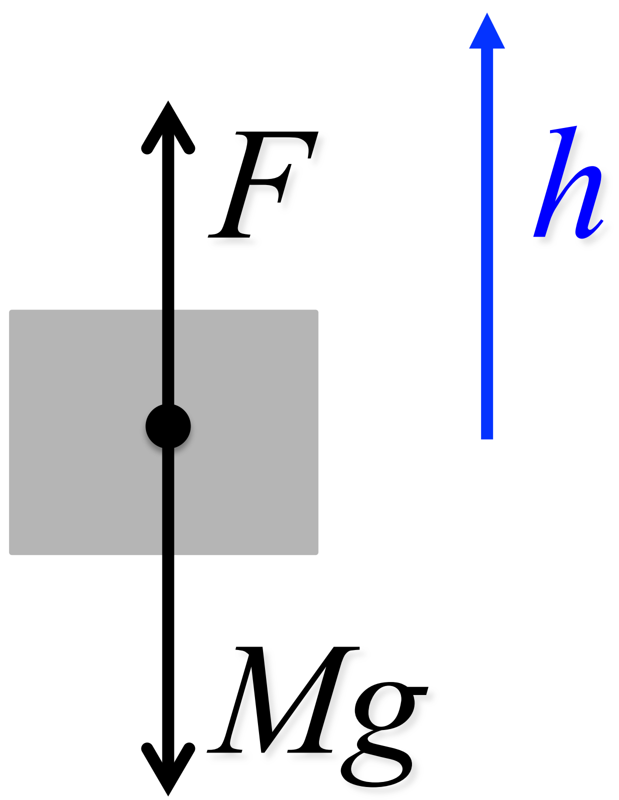
If $F = Mg$,

$$W_{\text{net}} = 0$$

Then $\Delta K = 0$

Work & Potential Energy

It turns out that one can define a 'Potential Energy', *U*, for ALL conservative forces as follows:



$$\Delta U = U_f - U_i$$

$$= -W_{\text{cons.}}$$

$$\Delta U_g = Mgh$$

$$= W_{\text{n.c.}}$$

Work & Potential Energy

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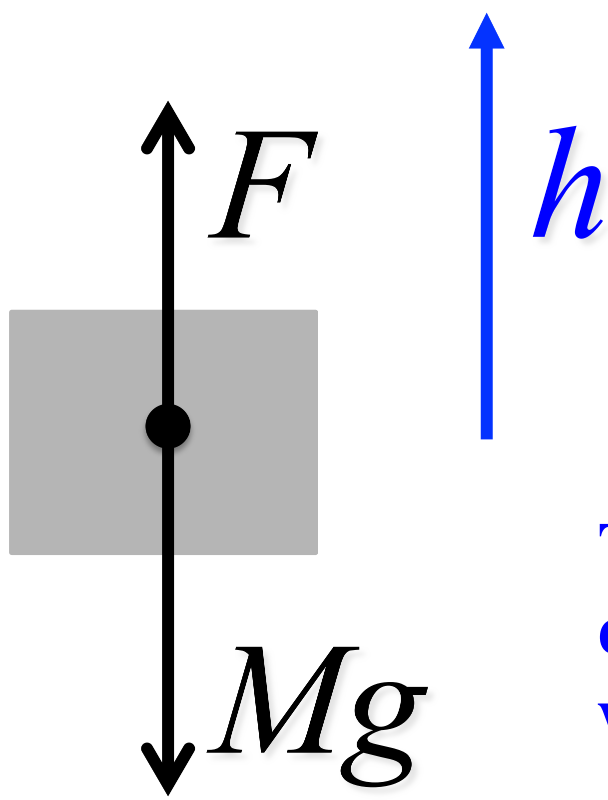


The Potential Energy change, ΔU, does not care how the height change was achieved.

Conservation of Energy

Work-Kinetic Energy theorem

$$\Delta K = K_f - K_i = W_{\text{net}}$$
$$= W_{\text{cons.}} + W_{\text{n.c.}}$$

- We can now replace any work due to conservative forces by potential energy terms, i.e.,

$$\Delta K = -\Delta U + W_{\text{n.c.}}$$

Or

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{\text{n.c.}}$$

- Here, E_{mech} is the total mechanical energy of a system, equal to the sum of the kinetic and potential energy of the system.

- If work is performed **on** the system by an external, non-conservative force, then E_{mech} **increases**.

Conservation of Energy

Special case: a completely isolated system, solely under the influence of conservative forces:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

$$\Rightarrow \quad \left(K_f - K_i\right) + \left(U_f - U_i\right) = 0$$

$$\text{Or} \qquad K_f + U_f = K_i + U_i$$

If there is an outside influence from a non-conservative force, then:

$$\boxed{K_f + U_f = \left(K_i + U_i\right) + W_{\text{n.c.}}}$$

Power

- Power is defined as the "rate at which work is done."
- If an amount of work W is done in a time interval Δt by a force, the average power due to the force during the time interval is defined as

$$P_{avg} = \frac{W}{\Delta t}$$

- Instantaneous power is defined as

$$P = \frac{dW}{dt}$$

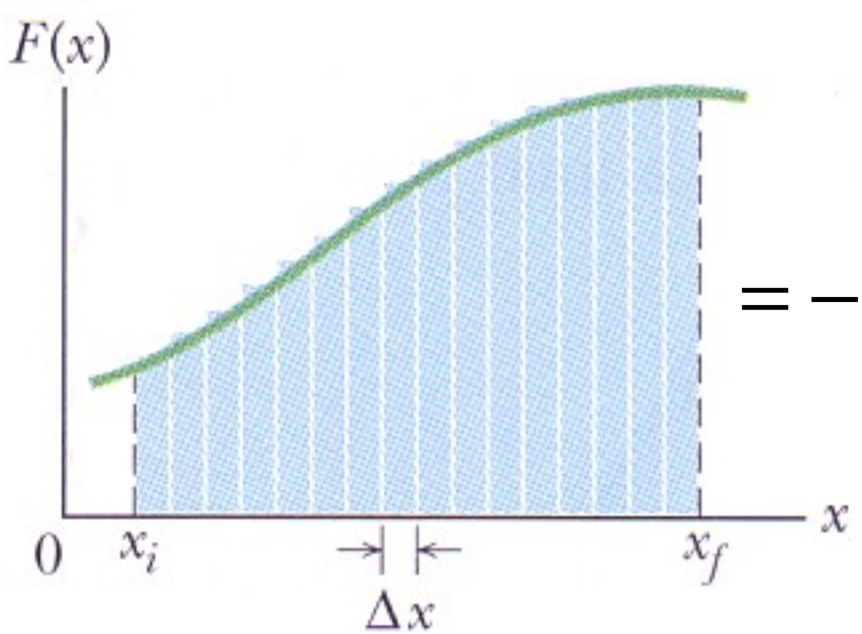
- The SI unit for power is the Watt (W).

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}$$

$$1 \text{ kilowatt-hour} = 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) = 3.60 \text{ MJ}$$

General (calculus) method for calculating Work

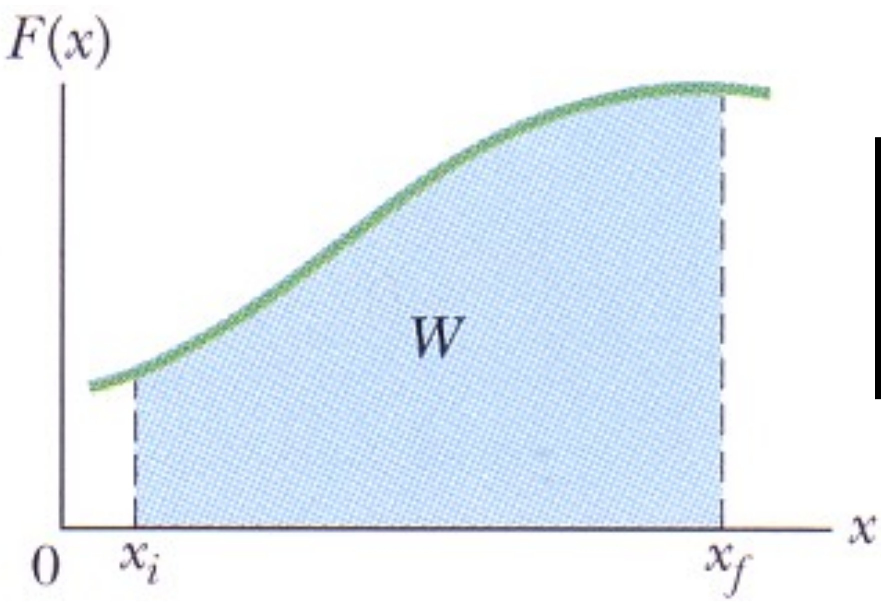


$$\Delta U = -W_c$$

$$= -\left(F_{c1}\Delta x + F_{c2}\Delta x + \ldots . F_{cj}\Delta x + \ldots\right)$$

$$= -\sum_j F_{cj}\Delta x$$

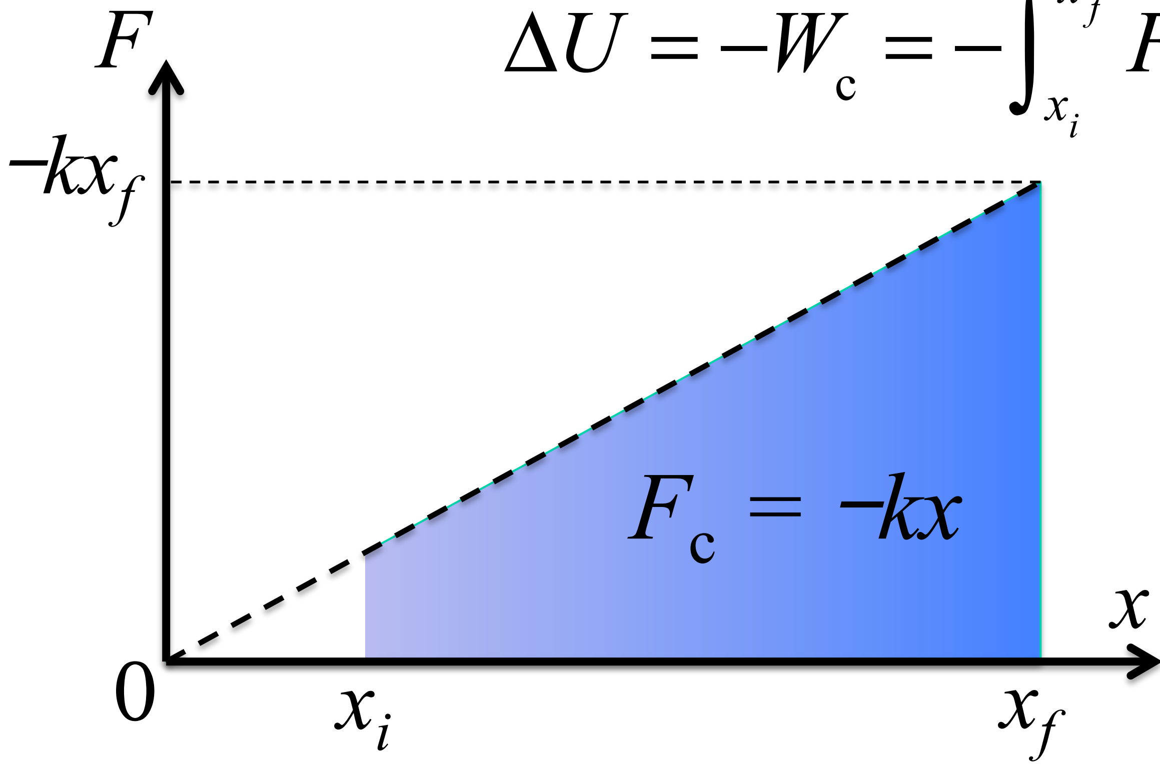
Take the limit $\Delta x \rightarrow 0$

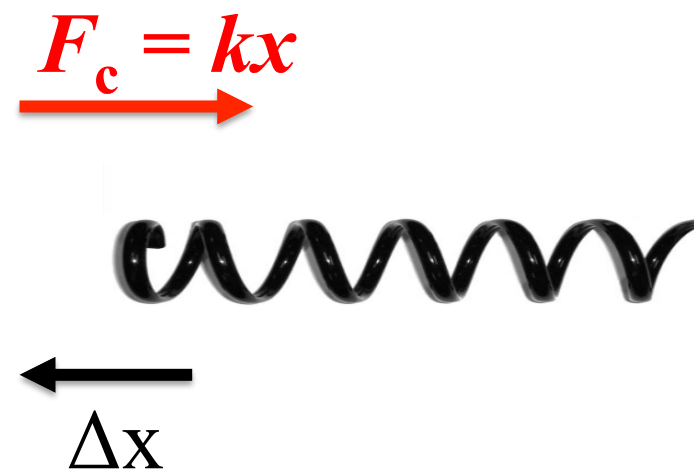


$$\boxed{\Delta U = -W_c = -\int_{x_i}^{x_f} F_c(x)\,dx}$$

Elastic/Spring Potential Energy

$$\Delta U = -W_c = -\int_{x_i}^{x_f} F_c(x)\,dx$$





$$U = -\int_0^x (-kx)\,dx = k\int_0^x x\,dx$$

$$= \tfrac{1}{2}k\left[x^2\right]_0^x = \tfrac{1}{2}kx^2$$